Number Sense or No Sense[1]

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Warm-Up Activity

The new fashion in Fiji primary mathematics classes is to begin with a warm-up activity. So even before I introduce my topic here is a fun exercise which will illustrate a few points in my presentation.

Paula had a large cage-like pen for his chickens. He could only see in it at the bottom and the top of the sides. That meant he could only see the chickens' legs and the tops of their heads. One morning there was a great commotion in the pen and Paula went to investigate. He thought there were more chickens than he owned in the cage because he could count 22 legs and 8 tops of heads. The extra legs and strange-looking heads were not his chickens' but belonged to pups. How many pups were in cage to account for the extra legs and heads? *(Answer: see page 21)*

When I did this with my students many of them were amazed that others would have such different strategies from their own. Then we did some other problem solving exercises including those in the pilot edition of Fiji Class 3 text. The discovery and the delight of my students at being given the freedom and encouragement to use their own strategies was great to observe. These exercises turned out to be an appropriate opening to investigating the sense-making nature of mathematics.

Among all of you here today, you probably used a variety of strategies to solve the chicken problem.

[These strategies were written up on the white board.]

Which do you now think was the best strategy? Here is a variation of the problem. Choose what you think is the most efficient strategy. This time there were 40 legs and 16 heads. *(Answer: see p 21.)* Was the strategy you used the most suitable for this set of data? Often we need to adapt the strategy we use depending on the numbers and context of the problem. This facility to adapt is possibly an indication of how well-developed our mathematical understanding is or, more specifically in this case, our so-called *number sense*.

What is number sense?

Number sense is not easy to define; it is easier to detect its presence or its lack of development. It is a delight to find a child who does a calculation in a way that shows insight into the number system. The now classic example of this is described by Carraher, Carraher and Schliemann (1987) who worked with the young street traders in Brazil. These children were able to do complicated mental arithmetic as they calculated the cost of the wares they were selling. Their strategies for calculating the costs were original and showed a great understanding of what they were computing. For example, a child was asked the cost of ten coconuts when the cost of one was 33 cents. The child said the cost of three was 1 cent off a dollar so the cost of 9 was 3 cents off three dollars and so another 3 cents and 30 cents gave a total of $3.30.

Last year I had the experience of what I suggest is an illustration of apparent poorly-developed number sense. Corpus Christi Teachers' College was having a *soli* (fundraiser). I was in the kitchen with

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a group of students who had the job of dishing up a specific number of dinners. When I saw them counting an irregularly arranged set of plates again and again as more were added, I suggested placing the plates in pairs to give 6 pairs per table (12 plates) so that the counting could be more efficient. Being very obedient CCTC students, they arranged the plates just so! But – when it came to the next time to count the plates the students proceeded to point to each of the plates one at a time as they counted.

Later, I sat with the Year 2 students who were aiming to reach a target of $1000 and had so far collected over $900. One student asked how much more they needed to reach the target. There was a call for pen and paper and someone proceeded to do the subtraction algorithm correctly. Then, when a further donation of $10 was passed in, another pen and paper subtraction was done. What amazed me, besides using pen and paper for that calculation, was the lack of embarrassment in doing this in front of their mathematics teacher of the past two years!!

Is having a well developed number sense important? Or perhaps a more basic question needs to be asked: *why teach mathematics in primary school?* Answers given to these questions include the following: *mathematics is needed in everyday living*, and *mathematics trains the mind to think logically.* But do the present courses do that? Think of adults who are not involved in any area of formal mathematics. The calculations they have to do in their day-to-day lives have been found to be mostly mental calculations involving estimations using approximations. (Wandt and Brown 1957). How often do they need to add or subtract two fractions or do a long division using the school-learnt algorithm?

Back to the question I asked earlier. What is number sense?

I would consider that *it is the ability and natural tendency to use and operate on numbers with understanding, efficiency and flexibility.* As I indicated earlier, we would find that people with a well-developed number sense apply their mathematics skills in day-to-day living, especially using mental computations and estimations.

McIntosh, Reys, and Reys (1992) have produced a framework to outline what they suggest is included in number sense. They divide number sense into three main areas – <u>number</u>, <u>operations</u> and <u>applications</u>. Within these areas there are a number of aspects. For example in <u>operations</u> there are two aspects:

- the understanding of and facility with the meaning and effects of operations For example, division includes breaking into a number of equivalent subgroups,
- the understanding and use of equivalent expressions which includes the ability to choose appropriate strategies for calculating.

How can we evaluate number sense?

After studying what others have done, I have devised a short-item test and a set of about 20 questions and tasks for an interview situation, based on work done by some mathematics educators (McIntosh et al. 1996; Kaminski 1996; Victorian Board of Studies in Melbourne 1998). Examples are:

- If the key for `5' is missing on a calculator, how could you use the calculator to do the addition 259 + 7634?

- Petero dropped a coconut from a tall coconut palm. About how long will the coconut take to reach the ground?

  1 sec.? 5 sec.? 10 sec.? 15 sec.?

- What mathematical working can be used to simplify the following?

  4 + 4 + 4 + 4 + 4 =

How many 35s can be subtracted from 456?

- How many digits are there in the numbers 1 to 100 inclusive?

- Think of the number 264. What could you add or subtract from this number to get an answer that has an 0 in place of the 6?

How can teachers develop number sense in their students?

Instruction programs specific to developing number sense are not often recommended. Instead, teaching is permeated with work that allows the students to construct their own mathematical knowledge with the help of discussion with the teacher and their classmates (Trafton 1989; Van de Walle and Bowman Watkins 1993). The modern jargon for this process is `social constructivism' and it generally includes number sense activities followed by discussion and reflection on them. As yet, and perhaps never, has the ideal mathematics unit for developing number sense been created.

We all have ideas about this, I am sure. At CCTC last year in my first year course I attempted to help the students' development of number sense using appropriate activities, followed by discussion and reflection.

One area of work the students found most helpful was the in-depth discussion of our place value system with its base of 10. The study of a base system other than 10, in our case base 5, and the production and use of teaching materials such as spear cards, the abacus and expanders specially used to illustrate base 10 were often commented on in their journals as being helpful to their understanding.

A large part of the reflection was done by journal writing in what we called "thinkbooks". The students wrote in them a few times a week and I read them and responded with a few written comments every couple of weeks. I feel that it was the actual thinking required in writing in the thinkbooks that was largely responsible for the development of their number sense. Some examples of entries:

Topic: Place value

Place value was an interesting topic for me. I did not know anything about place value before now ... I see the number 42 as four bundles of ten and 2 ones.

When we did place value I felt uncertain about it because I used to hate this topic in school. However, as we went on with the lessons I started enjoying it because I came to understand it better. I used to see 12 as just a number 1 and 2. It never occurred to me to think of 1 as one ten and 2 as two ones.

This semester I have come to know many things ... like place value. For this I am able to know how and why we place numbers in different places.

One area of work that students found most helpful and interesting was the mental computation strategies. A further quote from a thinkbook illustrates this fact.

If my primary teacher taught me all these small tricks I think and I am sure I wouldn't be poor at maths. I'll make sure I teach all these tricks to my class. I've learnt a big lesson. I was not good at adding these kinds of sums but now I know that I can do it.

Students' Presentations

The students did class presentations for most of the topics. Each week I spent time helping the groups of three that were doing the presentations the following week. Student presentations related to each of the topics took various forms such as warm-up activities, and demonstrations on the use of teaching materials. These presentations were very

beneficial to the students in helping to develop understanding and in learning teaching techniques but I have to admit that they were draining of my time and energy.

Research

As is true for most mathematics teachers I have spent my teaching career thinking about how to help students understand mathematics. Now, thanks to a grant from the Pacific Educators Development Scheme, teachers in the mathematics department at CCTC are motivated to research the question of how we can best develop number sense.

I am looking for answers as to how best to develop number sense of pre-service teachers. This coming semester I am going to evaluate formally my work with the CCTC first year students in their mathematics classes. My data for evaluations of students' number sense will come from pre-tests and post-tests, journal writing, questionnaires on attitudes and beliefs of mathematics learning and teaching, essays on past schooling, interviews and anything else that may provide helpful information.

I can easily list some of the limitations of this research –

- I am not a Pacific Islander,
- My sample is small with only 40 students,
- My own course is being investigated by me and therefore it will be difficult to be objective,
- I, the writer of the test, am testing the group I am teaching – teacher as researcher. It will be difficult for me not to teach towards the questions in my tests.

All this is a lead-up to an invitation to other maths teachers to collaborate with me in the research. Many of you were educated in the Pacific and have been working with students here for many years. You may have some answers I am searching for. People new to the Pacific, such as myself, have much to learn from your experience and I would be most happy to include your classes in the research if you wished me to do so.

Conclusion

I am impatient to find the answer as to how best we can develop students' number sense. True there is an urgency to find some solutions but I know I must accept the fact that the answers may not be forthcoming in a hurry and I may be looking for them in the wrong place or via the wrong method. If you have suggestions for me I would be most grateful to hear them. Possibly the best answers on how to develop number sense in the pre-service teachers will come from the students themselves if I listen and watch carefully. Then, perhaps, they themselves will provide the solution as to how best they can help their future pupils understand and apply mathematics concepts.

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