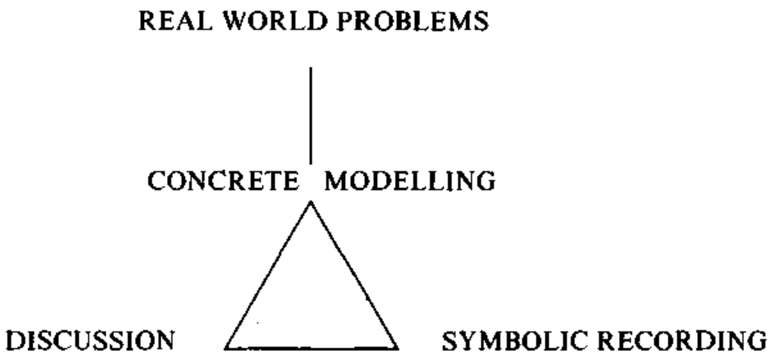
Teaching the Four Operations for Whole Numbers and Decimals

Michael Dunphy

It is largely accepted that teaching the four operations with whole numbers and decimals requires initial work with concrete materials, discussion between teacher and child for each phase of manipulation, and symbolic recording of the activity. This approach enables problems to be modelled with materials as the teacher through questions and discussion shows how to use the materials along with the symbolic recording for each phase of manipulation. It is contended that such an approach leads to understanding of algorithms i.e. symbolic recording of an operation. Teachers who begin with the symbolic recording of an operation without using concrete materials are depriving children of a high level of initial understanding. There is no need to teach these operations through a rote, mechanical, symbolic approach as this article demonstrates.

The basic reason for teaching arithmetic is that it is used to solve everyday problems. Since this is the case it is important from the outset that teachers use real-world problems as a basis for teaching the four operations. Such an approach enables students to see arithmetic in perspective.

The approach recommended can be summarized in the diagram below.



Initial work would progress from real-world problems, then to concrete modelling, discussion and symbolic recording. As children became competent with an operation they should be able to progress in any direction around the framework, and ultimately should be able to solve a given problem at the symbolic level. Thus, concrete materials are a 'means to an end', and help to provide students with a link between the original problem and the final symbolic recording.

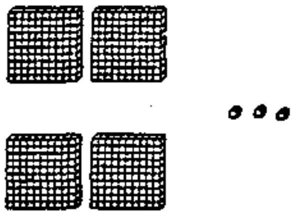
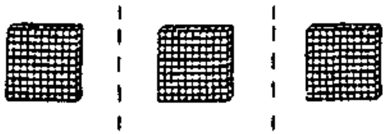
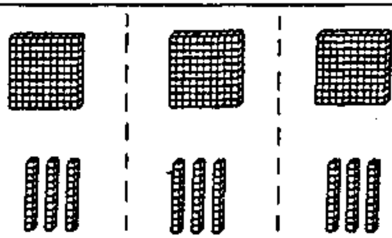
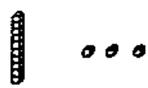
In this paper base 10 multi-base arithmetic blocks (M.A.B.) are used at the modelling phase. Although these can be purchased they can easily be made from cardboard. The pieces are:

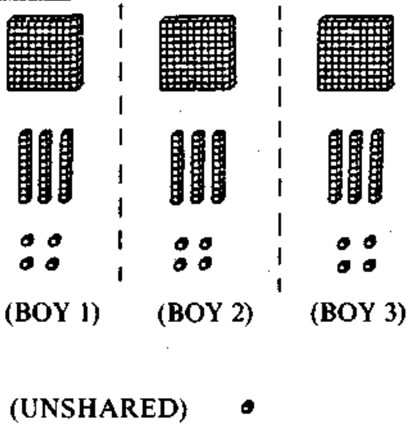
FLAT LONG MINI

Normally flats, longs, and minis would be used to represent hundreds, tens and units respectively. However, this can be varied according to the problem being modelled. This is particularly relevant to problems involving decimals. Teachers would ensure that initial problems were chosen so that they could be modelled with these three pieces. When students can perform an operation using M.A.B. with three digits the scope of the work can be extended to problems featuring more digits where M.A.B. is not used. Thus, M.A.B. provides a framework for computations involving three digits which serves as a basis for later work where M.A.B. is not used.

All operations can be taught using M.A.B. However, in this article division is used as an example. A convenient means of developing the division algorithm is through sharing or partition. The approach provides a sound basis for teaching the division algorithm using language with which students are familiar and provides a direct link to symbolic recording. The table below shows the relationship between concrete modelling, dialogue between teacher and student, and symbolic recording. The real-world problem for the development is shown below.

> 3 boys have 403 marbles to share. Find the number that each boy gets.

DIALOGUE	CONCRETE MODELLING	SYMBOLIC RECORDING
Teacher asks students to select M.A.B. to represent the number of marbles. Problem is recorded symbolically. (Known from earlier basic work with division.)		$3\overline{)403}$
Students share the hundreds; the teacher focuses on the number for each boy, the number used and the number remaining. Recording takes place as shown.	 (BOY 1) (BOY 2) (BOY 3) (UNSHARED) 	$\begin{array}{r} 1 \\ 3\overline{)403} \\ \underline{3} \\ 1 \end{array}$
Teacher asks students to convert unshared hundred to tens. These are recorded. The tens are now shared and the number for each boy, the number used and the number remaining are noted and recorded.	 (BOY 1) (BOY 2) (BOY 3) (UNSHARED) 	$\begin{array}{r} 13 \\ 3\overline{)403} \\ \underline{3} \\ 10 \\ \underline{9} \\ 1 \end{array}$

Unshared ten is converted to units. This gives 13 units overall, which is recorded. These units are shared between the 3 boys. Each gets 4 and this is recorded, as is the number of units used and the number remaining.		134 3) 403 3 10 9 13 12 1
On the basis of the above modelling and manipulation each boy gets 134 marbles and 1 is unshared.		

Students continue to use M.A.B. until they understand the basis of the division algorithm. At such a stage they work at the symbolic level only. The previous table shows a typical approach for completing any division problem. Problems involving division with double digit divisors and more extensive dividends are unwieldy to solve with M.A.B. For this reason it is necessary for pupils to have a good understanding of division with single digit divisors before they proceed to more complex problems. Although M.A.B. would not normally be used with such problems the sequence of commentary mentioned can still be maintained. This is shown in the solution of the following real-world problem.

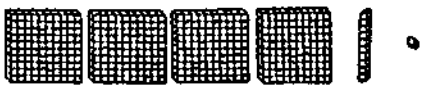
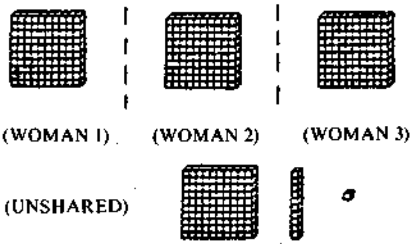
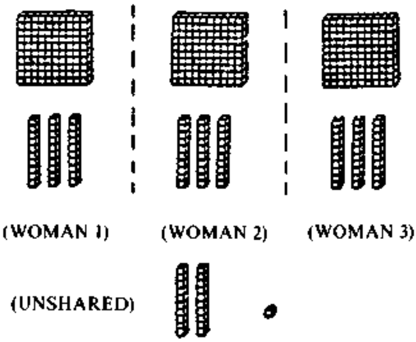
> 24 people win a total of \$3168 in Fiji Sixes. How much does each person win?

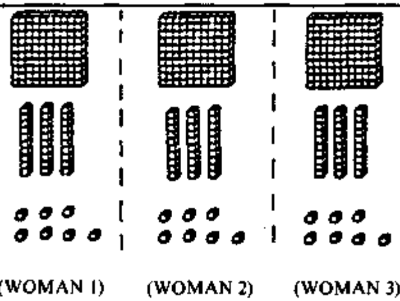
DIALOGUE	SYMBOLIC RECORDING
Teacher and students discuss the basis for recording.	$24\overline{)3168}$
3 thousands cannot be shared by 24 people. This is recorded.	$-$ $24\overline{)3168}$
Converting the 3 thousands to hundreds and using the 1 hundred gives 31 hundreds. Sharing between 24 people gives 1 for each with 24 hundreds used and 7 remaining.	-1 $24\overline{)3168}$ $\underline{24}$ 7
There are 76 tens overall. When these are shared between the 24 people each would get 3. 72 tens would be used and 4 remain.	-13 $24\overline{)3168}$ $\underline{24}$ 76 $\underline{72}$ 4
There are 48 units. Hence, each of the 24 people will get 2 units. All are used with none remaining. would be used and 4 remain.	-132 $24\overline{)3168}$ $\underline{24}$ 76 $\underline{72}$ 48 $\underline{48}$ $\underline{..}$
Each person receives $132.	

The method discussed can be extended to division problems involving decimals. M.A.B. can be used in initial examples and this will soon provide a framework for the solution of problems without the use of material. Consider the problem below.

> 3 women are to share 4.11cm of cloth. How much will each woman get?

In developing the model for this problem it will be necessary to use flats as units, longs as tenths, and minis for hundredths. The table below shows concrete modelling, dialogue and symbolic recording for each phase in the solution of this problem.

DIALOGUE	CONCRETE MODELLING	SYMBOLIC RECORDING
Teacher asks pupils to select M.A.B. to model the length of cloth. Problem is recorded symbolically.		3) 4.11
Sharing 4 units between 3 people gives 1 for each with 3 used and 1 remaining. This is recorded.	 	1 3) 4.11 3 1
There are now a total of 11 tenths to be distributed. Sharing gives 3 to each person with 9 used and 2 remaining.	 	1.3 3) 4.11 3 1 1 9 2

There are a total of 21 hundredths. Sharing these between the 3 women gives 7 for each. All are used and none remain.		1.37 3) 4.11 3 1 1 9 21 21 ..
Each woman gets 1.37m of cloth.		

The method described promotes student understanding of division and leads to a logical symbolic recording. Such an approach can be extended to the remaining operations. The method has relevance for both primary and secondary teachers. Primary teachers can introduce students to these algorithms where the use of concrete materials is so vital, while secondary teachers can make use of M.A.B. for pupils who need further assistance in mastering any of the algorithms. M.A.B. can be readily made, all that is required is a supply of cardboard. Clearly it is worthy of consideration since its use is likely to promote understanding.