Fruit Salad Algebra - A Fiji Experience

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The Background to the Workshop

Introduction

Algebra is frequently introduced with variables representing objects such as a for apples and b for bananas. The name 'fruit salad algebra' originates from this common introductory teaching strategy. This strategy has been used for a long time by many teachers, both experienced and newly appointed, in an effort to foster the important real world links.

At the end of every year students in Fiji sit for public examinations. Reports are compiled for each subject area highlighting students' performance. Within mathematics, algebra is invariably listed as being poorly understood and poorly performed (Ministry of Education, Fiji, 1991a, 1991b). Similar findings (as reported in Macgregor, 1986) appeared in a study of graduate students from N.S.W., Australia. He also notes that examiners of the 1984 Higher School Certificate found a lack of understanding of the fundamental ideas of algebra.

Some errors are reported more frequently than others in papers discussing students' interpretations of algebraic concepts. One of the more widely quoted is the 'professor-student' error. This is widely used as an exemplar (Booth, 1986; Macgregor, 1986; Philipp, 1992) and will be described here because of its importance. Thirty seven per cent of first year U.S. college engineering majors and fifty seven per cent of college social science majors were unable to solve the following:

> A university has six times as many students as professors. If S is the number of students at the university and P is the number of

professors at the university, then write an equation expressing a relationship between S and P.

Interviews were conducted with students who gave the answer as $6S = P$, in contrast to the correct answer of $S = 6P$. The data obtained from these interviews revealed that these students understood the variables S and P to be labels and not representing numbers (of students or professors). It is this error which forms the basis of this discussion. It leads to major problems. Some of these problems are now mentioned.

Errors made by algebra students

We selected some of the more common student errors reported in the literature. They include:

- Value depends on position in alphabet. Therefore $y > a$ (Milton, 1988).

- The tendency to *close* (Milton, 1988; Booth, 1986). Prior to meeting the concepts of algebra, students have had a great deal of experience of getting a single number answer, such as $4 + 5 = 9$. So there is a strong motive to write $4a + 5B = 9ab$. More evidence of this is given by students who change the expression $2 + x$ to $2x$ justified by the thought pattern *43 = 4 tens + 3 units* or $2 + \frac{1}{2} = 2\frac{1}{2}$ (Booth, 1986; MacGregor, 1987).

- The letter must stand for something beginning with that letter (Booth, 1986). y must therefore be *yacht, yam or yoghurt.* This is reinforced when we use formulae such as *area = length x breadth (a = l x b).*

- Each letter has a specific number value and $x + y + z$ could never be equal to $x + w + z$ (Booth, 1986).

- From the beginning, students are told not to add unlike terms. Again, there is a tendency among teachers to use objects to stop this tendency to close. The strategy has the unfortunate side effect of reinforcing the notion that letters must stand for something, rather than a number or series of numbers (MacGregor, 1987).

The difficulty students have with the concept of a variable may arise from the many uses of algebraic variables (Philipp, 1992; Schoenfeld & Arcavi, 1988). Students have to choose, for example, whether the literal symbol is taking the role of a parameter, a constant, an unknown, a label, a generalised number or a variable.

Schoenfeld & Arcavi (1988) remind us how easy it is to ride a bicycle after we master the technique and after many, often painful mistakes. They suggest that algebra has many pitfalls in its learning which we, those who have mastered the techniques, need to be kept aware of in order to understand the difficulties of the new learner.

Possibilities offered by the literature

The literature offers several alternative strategies for introducing students to algebra. We discussed the implications these might have for the mathematics classroom in Fiji. The confusion with x^2. To be able to give students the knowledge necessary to successfully manipulate variables seemed a positive and achievable target. The trials and comments we read, gave us the confidence to spread these findings among our fellow teachers.

The pressure of public examinations on teachers in Fiji is enormous. A teacher's professional future and the reputation of the schools in which they teach, strongly relies on the results which are publicly printed in the local press. This pressure entices teachers to encourage students to rote learn theory and solutions to problems from previous examination papers. The potential offered by the literature for understanding, was indeed exciting.

Clear evidence of the many difficulties experienced and the possibilities offered to improve the teaching of algebra encouraged us to share this information in a workshop situation. We discussed the importance of getting a group of teachers together to talk about introductory teaching strategies and to present some possible alternatives. This led to the planning of a one-day workshop based on the theme of presenting introductory algebra. Teachers of Forms 1-4 mathematics were targeted. In Fiji, Form 1 students have completed six years of primary schooling. A total of 16 experienced teachers from the Suva-Nausori area participated. Course

presenters came from a range of backgrounds. They included staff from the Fiji College of Advanced Education (FCAE), the University of the South Pacific, the Curriculum Development Unit and local high schools.

The following sections report on the commonalities in teaching algebra which arose during the workshop and the teachers' reactions to the ideas presented.

Design and implementation of the workshop

Workshop design

Many schools in Fiji are faced with the problems associated with recruiting enough qualified and experienced teachers of mathematics. Even if these staff are secured, they are usually involved with the senior forms (Forms 5, 6 and 7). The problem has become more acute in recent years. A consequence of this is that students in Forms 1-4 lack the necessary quality learning environment in mathematics. From our observations we learnt that their teachers are frequently inexperienced, have little or no teacher training and inadequate subject area qualifications (many completing only Form 7 level of education). The workshop was planned as a venue where teachers could discuss and share strategies which they currently used and to offer opportunity for comment on, and discussion about, strategies and ideas introduced during the workshops.

The local examiners' reports and mathematics research papers frequently report that students at all levels of the education system continue to have difficulties with algebra. Such difficulties affect performance in many other areas of mathematics. The workshop was planned to give teachers an increased repertoire of approaches which might be used during the initial stages of algebra. It was designed to help teachers become more comfortable with algebra by having a greater awareness of alternative strategies available to them and a better understanding of student reaction to algebraic information.

The workshop would involve sessions targeting the introduction of algebra to junior high school pupils. Following a session in which the teachers were able to discuss their current methods of teaching algebra, ideas for different methods of introducing algebra were to be presented. We were enthusiastic that teachers would recognise that these methods, in contrast to the traditional 'fruit salad' approach, had the potential to help reinforce the concept that variables represent numbers rather than objects.

We intended to introduce this approach to a group of graduate (mostly BSc from the University of the South Pacific), experienced teachers who were to attend the final stages of their in-service teacher training certificate course at FCAE.

Comments on the text approach

Students throughout Fiji are exposed to common texts, curriculum and external examination. It is therefore useful to have a brief look at the way students are introduced to algebra via the text. We must bear in mind the level of teaching expertise of some lower form teachers. Though often enthusiastic, these teachers have little training in alternative methods of presentation of material and tend to adhere to the methods shown in the text.

The curriculum guidelines show that algebra is not formally introduced until Form 3. In Fiji this is part of the problem. Listed here are several of many instances where algebraic concepts are merely put to students as facts to be absorbed prior to any structured introduction. These isolated (i.e isolated from follow-up and explanation) facts begin to appear as early as Form 1. Students then have a great deal of time to internalise their own system of algebra before it is presented to them in any systematic manner.

How the text introduces algebra in Forms 1, 2 and 3

The structures which follow are copied straight from the texts. There is strong evidence that the texts make little effort to introduce students to the new language and concepts which form the basis of the algebraic structure. There are many other instances in the texts of algebra usage appearing

before its formal introduction. One instance has been selected from the Class 7 text and one from the Class 8 text. Class 7 and Class 8 students are in their 7th and 8th year of schooling respectively and represent the upper primary in some schools and the lower secondary in others. The words and examples which follow have not been isolated from a context of long and descriptive passages with plentiful examples. Explanations of techniques and terminology in the text are as they appear here.

Class 7 text (equivalent to Form 1)

This text contains a great deal of algebra. In reality students begin their algebra here. They are given as facts to remember. No whys or hows are put forward. Teachers and students have begun algebra without knowing or recognising it. No wonder there is confusion when statements such as the following appear.

> The rule is r = 2x (i+3)
> Mathematicians normally shorten this to: 2(i+3)

Class 8 Text (equivalent to Form 2)

With no introduction to the concept of a variable, students are posed the following:

> Complete a table using $p = n^2 - n + 41$

Mathematics Book 3B (for Form 3)

This is where the concept of algebra is formally introduced. Notice how this is done. Students are given the following with no lead into it.

Addition and Subtraction of Monomials

Write each of these more simply where possible

a. 3 dogs + 2 dogs	e. 6 cats + 4 dogs
b. 9 cats - 3 cats	f. 3 cats - 2 dogs
c. 3a + 2a	g. 6a - 3b
d. 6x - 4x	h. 7x - 2y

Cats and dogs indeed!

A review of the participants' current strategies

Teachers attending the workshop had been teaching from three to twenty years. Although Forms 1-4 teachers were the target group, we found about half to be senior members of staff. This unintended mix actually gave us a clearer picture of the stage algebra teaching in Fiji had reached. Workshop participants were divided into five groups and given large sheets of paper. They were asked to prepare group answers to the posed situations.

How would you teach the following problems to school students who had little, or no, knowledge of algebra?

$2a + 3a =$

$3a + 2b =$

We emphasised that we wanted to see a range of strategies from teachers from varied backgrounds and experience. There is no one correct method. Completed sheets were hung around the room for comparison. We, the authors, saw what we had expected: a tremendous uniformity of approach. Apples, bananas, cats and dogs, and absolutely nothing else.

The results indicated that lessons planned by the groups were closely linked to the presentation shown in the textbook. Later, during the group

discussion sessions, it became evident that they too had been exposed to similar strategies when they were at school and when they observed fellow teachers. This narrow approach to introductory algebra may be a result of a lack of training in ways to develop alternative lesson plans.

We later ran this workshop with a group of teachers attending an in-service summer school. This group had a similar range of experience as the first group. The results were identical: cats, dogs, apples and bananas.

Presentation of ideas to workshop participants

Technological development in Fiji is in its early stages. This prevented our looking at technology based solutions to the problems of teaching in this area.

Discussion of new ideas centred on the points mentioned in the next few paragraphs.

Many texts treat the introduction of algebra with a mechanical definition which may read something like: "A variable is a symbol used to represent an unknown number (or numbers) until such time as its value can be determined." Students may be given facts such as this and left to incubate their own interpretation, frequently developing incorrect concepts of the symbolism involved, its meaning and its manipulation.

Students need time to develop the concept of a variable. They must progress from dealing with numbers and symbols with one answer to thinking about a generalisation with many possible answers for the same expression. We must use ideas within the students' own reality to help them internalise this transfer (Booth, 1986; Briggs et al., 1986; Philipp, 1992; Schoenfeld & Arcavi, 1988).

Without sound groundwork, students will have a great deal of trouble with the higher levels of algebra. This lack of understanding of basics is evident even in students who choose to study mathematics at the tertiary level.

Highly acknowledge among alternatives to 'fruit salad' are strategies which involve students in activities which lead them in steady progression to early algebraic concepts. Discussion among students about problems which have several methods of approach and/or several solutions form the basis of much of the learning process. This directs the learner to a recognition of the generalised nature of algebraic expressions and equations. This is the real power of algebra (Booth, 1986, 1989; Briggs et al., 1986), forming a foundation for many further studies in mathematics.

Researchers recognise that students need to progress within their own intellectual and experiential bounds (Milton, 1988). Time to absorb enough information to enable the abstract manipulations to be performed is required (Booth, 1989; MacGregor, 1987; Milton, 1988; Pegg & Redden, 1990).

Milton (1988) and Pegg & Redden (1990) recommend that teachers spend time emphasising the pattern and structure found in arithmetic. They should then use this structure to build activities which become more generalised in their approach and in the results achieved.

This is extended by Booth (1989) and Pegg & Redden (1990) who suggest that students explore series of geometric patterns to look for written solutions which describe the sequence shown. The important thing is that students discover, discuss and write about their solutions. Using a written system to describe the first three shapes in a series may be simple, but how about the 100th shape? They need to be guided to the conclusion that a system which helps make all the writing more concise is both necessary and time saving. Barbeau (1991) uses a careful number sequence approach to bridge between arithmetic and algebra.

Time needs to be made available for students to discuss the many roles literal symbols may take (Philipp, 1992; Schoenfeld & Arcavi, 1988). Briggs et al. (1986) advise that this time taken in trial and error and discussion will give other benefits, such as significantly improving students' problem-solving skills.

To help students overcome the tendency to think that letters should represent things beginning with that letter, we must take care in the presentation of formulae. Maybe we could swap *p* or *t* or *h* for *a* when discussing area. Booth (1989) supports the notion of encouraging students to use their own symbols or letters. This makes their answers more personal and therefore more meaningful.

Booth (1986) also pointed to the importance of emphasising the two way nature of the '=' sign as meaning 'is equal to' or 'is equivalent to' instead of the often used 'makes'. This will reinforce the bi-directional sense of the sign and demonstrate that it is unnecessary to close all solutions.

Variety is seen as another key to effective learning in algebra. Geometric constructions and the use of paper folding are described in Sobel and Maletsky (1988: 155-157 and :171-176).

Many more strategies appear in the references listed. Those selected highlight the extensive array of alternatives teachers have at their disposal. Incorporating these into their own teaching will help students to overcome the difficulties experienced in the learning of algebra.

Teacher comments on the workshop and the material it contained

All participants filled in a prepared evaluation form at the end of the workshop. Particular emphasis was placed on obtaining feedback on how they perceived the new ideas for teaching algebra. A discussion followed.

All teachers agreed that a variable in an algebraic equation should be treated as a number rather than as an object. The result is very significant because at the start of the session all teachers stated that the initial stages of algebra teaching would use objects to demonstrate the use of literal symbols.

One teacher commented that he would now use *a* as representing a number, rather than apple in expressions such as $3 \times a + 2 \times a$ "because it would be of help in future when we talk about for example $3a + 2b$".

Another teacher reflected on the difficulty his students had in relating algebra to objects. He considered that the new approach would be "a very useful approach to use".

Trialing is important in assessing the suitability of any new strategy that may be used in local schools. This fact was recognised by a teacher who wrote "I will try to use the same approach in the classroom and then get feedback from my students and if I find that it works then I'll change my methods which I used to follow..."

The significant aspect of the evaluation was that teachers apparently appreciated the constraints of the 'fruit salad' approach and that they were willing to try out alternative ideas. In the context of the teaching methods used in Fiji (frequently straight from the text), this was seen by the organisers as a significant and positive response to the material contained in the workshop.

Conclusion

There is strong evidence that traditional algebra leaves students confused. The 'fruit salad' (or zoological, if we insist on cats and dogs) approach has been shown to be weak in its theoretical argument. We, the teachers of algebra, need a change.

The readings cited suggest that algebra must be introduced in a manner which will enhance student capability to build on their knowledge when more advanced topics are reached. The foundation must be firm. While algebraic concepts are introduced to students as early as Form 1 in the upper primary/lower secondary school, it is not until Form 3 mathematics that these concepts become a formal part of the curriculum. Questions arise as to the correct timing of the introduction of algebra and how it should be done.

The workshop signalled that it is important for teachers to keep in touch with new developments in their subject area and that the text they are using should reflect these developments. In the Fiji context this means the

textbook must surely be modified to cater for the new ideas in preference to the 'fruit salad' approach to the teaching of algebra.

For classroom practitioners, it is necessary to initiate changes in strategies used to teach algebra. This has wider implications for the whole of the mathematics curriculum. Further in-service courses should be implemented as a priority to target larger groups of teachers. The Fiji Mathematics Association, the University of the South Pacific, the Fiji College of Advanced Education, and the Curriculum Development Unit are ideal bodies, institutions or sections to take up such initiatives.

Appreciating the need for action, the Curriculum Methods courses at the FCAE have already been modified to include these ideas for pre-service and in-service teachers attending the College.

This workshop and discussion provided strong evidence that we should look at the texts we use and check whether they meet the criteria suggested for developing new approaches to the teaching of this important topic in mathematics.

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