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The Language Factor in Mathematics Education[1]

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Language is an important factor in the learning and teaching of mathematics. While for most students a mathematics lesson is generally a language lesson within the mathematics part, the sequence seems more complicated for second-language learners. For many Pacific Islanders, learning in English and Mathematical English creates serous cognitive difficulties. Using appropriate language(s) in context must be a consideration.

Introduction

Towards the end of last year (1998), a few weeks after the Fiji national examinations, some letters appeared in the Letters to the Editor column of the Fiji Times concerning the mathematics examinations. Two in particular are relevant – firstly, one which came from a Form 6 Fijian student, and secondly, the reply to it from someone who had taught mathematics.

In her letter "Maths Exam" (FT 3/12/98), the student complained that the sixth form mathematics examination was "full of words". She deemed the word problems unfair, and seemed to think that "wordless problems" would have been better and would serve the same mathematical purpose. A week later, a reply to this complaint appeared in the same column. The second writer expressed his disappointment at the student's comments and argued that, as a tool, the value of mathematics is in its use to solve the "word problems" that come with real life situations. While he acknowledged the language difficulty of second language learners, the writer insisted that making meaning of mathematical based problems rather than mechanistic 'monkey maths' must be the aim of mathematics

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learning. He attacked teaching which did not generate understanding.

While I concur with the latter's suggestions about meaningful mathematics teaching, I do not think that he fully appreciates the frustration and stress of *learning* in a second language generally, and mathematics in particular. As a Fijian who learnt and taught mathematics in English as a second language, I believe there is a case to answer. In this paper I explore the value and critical role of language in making sense of mathematics. I begin with an analysis of the nature of the language of mathematics, and then highlight some linguistic aspects in mathematics learning that will help us to appreciate the extra burden borne by second language learners.

The language of mathematics

Mathematicians have for long made us believe that mathematics is a neutral entity that is culture-free, that its concepts are the same everywhere. This culture-free perspective has been challenged strongly over the years, particularly by indigenous groups who have claimed marginalisation in mathematics learning. Bishop (1988) clarified the relationship between culture and mathematics and expressed the view that mathematicians confuse the 'universality of truth' of mathematical ideas with the cultural bias of mathematical knowledge and expressions. He defines mathematics as a cultural product that has a built-in context. This has important implications for language, to be discussed later.

Mathematics is also a powerful language of communication. It is a kind of hybrid language that is made up of ordinary English (OE) and mathematical English (ME), and matched with the western view of the world. Mathematical English comprises the mathematical register and the language of symbol notation. According to Pimm (1991), the mathematical register that comprises the technical terms of mathematics developed largely from ordinary English. The practice in its development has been to redefine simple, ordinary words rather than coin new ones. This has resulted in:

i. "multi-meaning" words which have a simple meaning in natural language, but also a precise, more complex mathematical definition, symbol and use. E.g. set, root, function.
ii. "word combinations" which carry different meanings from the sum of their everyday use. E.g. pie chart, simple interest, square foot. (Halliday 1978)

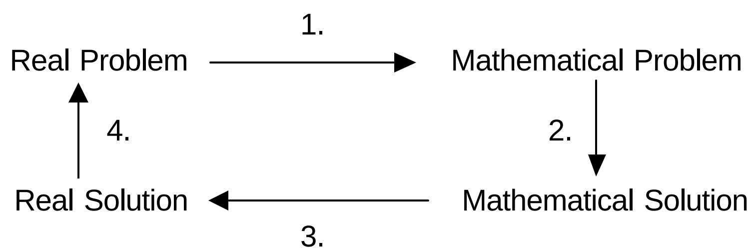
The symbol system is mainly separate from that of natural language because it uses a different, more complex rule-governed writing system. Schweiger (1994) locates it also as mostly based on a western heritage, even though the symbols are mainly non-alphabetic, having being formed through a complex form of encoding. He traced some symbolic symbols to the Roman, Greek, Hebrew and other western traditions. Others are believed to be of Asian origin, particularly Indian. There are more types whose origins are not as obvious.

Symbols are characteristic of mathematical texts and are also a powerful aspect of mathematics. For many students, however, symbolic representation contributes to a great deal of confusion in mathematics and, as Pimm (1991:20) suggests, it is "the very `concreteness' of the symbols and the absence of obvious mathematical referents [that] can lead many pupils to believe that the symbols *are* the mathematical objects".

The problem with mathematical word problems

> Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language. (Durkin and Shire 1991:3)

Many mathematical tasks are presented to students as word problems. Like the ones that the student complained about in her letter, word problems comprise a couple of sentences giving pieces of related information, and it is the student's task to perform appropriate mathematical operations and supply the answer. Where word problems are drawn from real life situations, the task in learning is to decode the written language into mathematical language, solve it mathematically, and then reverse the processes. Begg (1998) considers a typical mathematics problem in four stages:



In the diagram, step (1) or 'problem comprehension' is about translating each sentence of the structure (concept formation), and, like step (4), is overtly language based. Step (2), which involves using a learnt algorithmic procedure, has now been taken over by computers and calculators. This leaves steps (1), (3) and (4) that involve 'translating a real problem into a mathematical model, reinterpreting the mathematical solution back into the real context and checking the reasonableness of the solution in the real world' (Begg 1998:5) as students' tasks. The critical point here is that what students comprehend from the words of the question help them decide on the mathematical strategy to adopt. Research suggests that, very often, incorrect answers for word problems are due to students performing "correct arithmetic computations on incorrect representations of the problems" (Lewis & Mayer 1987: 364).

Miscomprehension of the information mainly in the form of 'relational sentences' and 'compare problems', and the use of logical connectives are common language problems in mathematics problem-solving. Relational sentences include three term series problems or transitive inferencing problems (Galligan, 1997) such as: "Tevita is heavier than Jone, and Manasa is lighter than Jone. Who is heaviest?" The two statements suggest a particular ordering of the three terms, Tevita, Jone and Manasa, which is arrived at by the integration of the two individual two-term arrangements arising from each of the two statements, into a single three term arrangement.

'Compare statements' are those that define one variable in terms of another. E.g. "There are six times as many men as women." "He is half as heavy as his son."

Another type of 'compare statement' is represented by these two different representations of the same problem.

i. Jone has 5 marbles. Tomu has 8 more than Jone. How many marbles does Tomu have?

ii. Jone has 5 marbles. He has 8 marbles less than Tomu. How many marbles does Tomu have?

Research has revealed students' difficulties with integrating statements into single terms, as well as sorting relational statements of word problems. It seems that problems that have these unique combinations are more difficult to solve than those that do not.

The difficulty for second language learners

Mathematics education is undergoing change both in form and focus. Change in our understanding of how people learn has shifted focus to students' thinking processes. The importance of students making sense and conceptualising has led to the concern for increased student participation in learning, including their ability to discuss, explain and clarify their ideas.

In mathematics, I place language as the most obvious difficulty which our students encounter. The unique register and special written form of the language of mathematics could cause many students of mathematics, monolingual English language background students and second-language learners alike, to experience difficulty with the language of mathematics texts and discourses. It has been suggested that for most students a mathematics lesson is generally a language lesson within the mathematics part. However, the sequence seems more complicated for second-language learners. For us, mathematics has always been taught in English. For the majority of our students, English is a second language, and for many others it is a third and not encountered until the child begins school. For all of us, having to learn in a language which is alien and often not reinforced outside the school is a serious handicap. In class, students encounter problems of understanding the teacher and the material, and also experience the stress of struggling to express themselves in English. Whereas it has often been suggested that our children rarely talk or communicate in class out of respect for the teacher, I argue that a lack of facility with the English language and the shame of poor articulation are greater barriers that hold them back. Looking back on my experience as a mathematics teacher, I remember well the difficulty of helping students make meaning of mathematical problems, particularly at the upper secondary level, where mathematics is more abstract. Some concepts just never registered. My colleagues and I followed the text closely for fear of teaching the wrong thing. Fear of misrepresentation often restrained us from straying away from the given statements and exercises (foreign as they were). When students found it difficult to grasp a point, we faced the problem of finding alternative ways of expressing it while keeping the meaning as close as possible to the original. In the end, against our best intentions, many resorted to drill and practice methods.

Howard (1995) has proposed that the demand on indigenous students coming from a cultural background very different from that of the school is heavier than that borne by students whose culture and language is closer to that which dominates in the school. This is because indigenous learners have to:

- adjust to the culture of the schools
- meet the challenges of a wider community culture
- develop their knowledge of the school language
- become conversant in English, and
- learn the language of mathematics including words, signs and symbols. (Howard 1995:9)

Verbalising algorithmic steps in solving problems is important for clarity of thought and understanding. In the mathematics classroom, our students have not only to verbalise in a second language, English, but also they face the additional difficulty of mastering the special register of the language of mathematics.

Possible way forward

I began this paper with a discussion of a Fijian student's anguish over word problems and the difficulty of learning in a second language. I propose that improvement in linguistic skills as well as the development of local contexts of learning could pave the way to greater understanding.

Teacher education must help mathematics teachers not only to know the content areas of mathematics but also the contexts, including the linguistic resources involved. To be able to understand the mathematical text, the reader has to have a context for it and thus be able to consider the technical terms correctly and see the relationships. This includes some understanding of English and accepted patterns of reasoning in that culture. In schools, mathematics teachers have to be language teachers and teach the language of mathematics as an integral part of mathematics. They could ask their language counterparts to spend time on the linguistic skills involved constructing 'relational sentences', 'compare statements' and logical connectives that are important in mathematics. It is also important to understand that our students' miscomprehension has much to do with the way they verbalise in the first language whose structures are different.

Local contexts – ethnomathematics

All use of language has a context (Halliday 1989). Halliday gives a useful discussion of the importance of context for understanding. He perceives the context as 'a kind of bridge between the text and the situation in which the text occurs that enables the reader to understand the text and interpret its meaning . . . If the reader or listener is not able to bring to the passage appropriate assumptions derived from the context of situation, then there is no information and no communication'. For our students, this leads to frustration and possible miscomprehension.

I believe that greater emphasis needs to be put on the context and application of problems. We need to ask ourselves the questions: How much of the mathematics we teach has application to students' reality? Whose reality is contained in mathematics texts? For the Fijian student in question as with others like her, learning in context is important for concept formation and for the context be relevant and meaningful. The identification of mathematical ideas in different cultures which come under the study called ethnomathematics suggests that there are 'other' mathematical systems determined by different languages and culture modes (Barton 1997; Gerdes 1995; D'Ambrosio 1995). In my research project (Bakalevu, 1998), an analysis of Fijian society from a mathematical perspective highlights Fjiian mathematical ideas and ways of mathematising that are different from western ideas and ways.

Fijian ways of mathematising

Fijian students come to school from a society that has its own ways and means of mathematising. While there is no category 'mathematics' in Fijian culture, there are mathematical ideas embedded in aspects of the socialisation processes and activities of the people. In my project, I focussed on the systems of counting, measuring, economics, navigation, kinship and design, all of which depict people's ways of solving problems. They are, in the main, different from the systems in the school and in mathematics.

Some important findings of the investigation suggest that:

- the Fijian is not prone to quantifying to the same extent as required in mathematics,
- Fijian methods of numeration suggest that Fijian students may have a limited mental map of the world of numbers and

operations, as well as a less well-developed number sense than most others,
- Fijian students may have a limited conceptualisation of formal measurement units,
- the Fijian system of counting certain items in tens could be useful context for developing place value, decimals and percentages etc.,
- there are numerous Fijian descriptors that are more commonly used than numerals,
- Fijian contexts of *dinau* and *kerekere* (redistribution and reciprocity) make sense to the Fijian (even if the rest of the world thinks they are not economically sound) and could be used to establish the ideas of negative numbers (normally difficult to visualise) , profit and loss, percentages, multiplicity, factors, etc.,
- there is a lack of a full mathematical lexicon in the Fijian register,
- the concepts of estimation and approximation are part of everyday transactions and pronounced extensively in a descriptive language,
- The process of mat weaving has 'hidden' mathematical skills while the more obvious concepts of periodicity, symmetry and tessellations are easily demonstrated.
- Tapa printing presents an excellent context for most geometric transformations.

These findings are important in that they are the ideas that Fijian children live with, understand and value. The contexts also have the relevant linguistic resources. According to constructivist ideals, they are the ideas that students will hold onto and retain longest. The challenge for mathematics teachers and educators is to plan activities in contexts which enable students to make connections between the mathematical ideas in their culture, and those in the mathematics classroom and the broader culture. Systems need to be in place to assist mathematics teachers to do that effectively. In support of this programme, Barton (1997) makes the point that indigenous learners who know two mathematical systems, speak two languages and have been introduced to two ways of knowing and seeing are better off than those with a monofocus. It makes perfect sense to encourage and support them to propogate both systems.

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